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Hybrid Control Scheme for Maneuvering Space Multibody Structures

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A hybrid scheme is proposed for the configuration control of a space multibody structure actuated by internal forces. The hybrid controller results from the combined action of a continuous inner loop and a digital external one. The continuous feedback is designed to obtain a computable sampled model; then a digital multirate control strategy is computed by inverting the input-to-state map to achieve the desired orientation. Simulation results are presented to emphasize the performances of the control scheme.

I. Introduction

STARTING from the firsts results in the 1980s, nonlinear methods today provide a powerful set of tools for solving control problems in several applicative domains.[1,2]

Configuration control of articulated orbiting structures is an appealing case study for the more recent control techniques. Actuation by internal forces and torques represents a nonholonomic constraint on the dynamics that render the problem not directly solvable by either classical control methods or more recent nonlinear approaches.[3,4] In fact, in this case the dynamics is nonlinear without controllable linear approximation, and nonlinear inversion-based methods fail because of the presence of zero dynamics, which is not asymptotically stable.

A major contribution to the control of nonholonomic systems came from the research on wheeled mobile robotics: the absence of sideway motion for a wheel represents a nonholonomic constraint. The relevance of the problem in mobile robotics stimulated a wide research effort, which has given most of the results that are known today.[5–8] The approaches proposed for such a class of systems are based on the so-called kinematic model, i.e., the differential relationship between velocity and position variables.

In spite of their relevance in several real cases, the control of nonholonomic dynamics in space applications remains little investigated. A first approach to configuration control of multibody space structures can be found in Refs. 9 and 10, where an open-chain mechanical structure composed by bodies linked by revolution joints was considered. A structure composed of three links was studied in Refs. 11 and 12, where different design procedures were proposed starting from the dynamic model and the kinematic one, respectively. A two-step design procedure is presented in Ref. 11 by separating the actions for achieving the desired configuration and orientation, while in Ref. 12, following the approach proposed for mobile robotics, a control strategy from the velocity variables is computed.

Following the approach proposed in Ref. 11, more recent papers[13,14] studied the nonplanar problem, with more emphasis on the modeling difficulties.

In this paper it is shown how the digital control approach proposed in Ref. 15 for solving the motion planning problem can be extended to the control of articulated structures. The general design procedure as proposed in Ref. 16 consists of a preliminary continuous feedback to get an exactly computable sampled model, followed by a digital controller. The resulting control law is piecewise continuous. The feasibility of the design procedure results from a suitable relationship between the kinematic and dynamic models of nonholonomic mechanical structures. With respect to the solutions proposed in Refs. 11 and 12, our controller provides the torques for achieving simultaneously the desired configuration and orientation. Moreover, with respect to the inversion-based design technique proposed in Ref. 11, our control strategy admits an iterative implementation that is viable in presence of perturbations and model uncertainties.[17]

The paper is organized as follows: In Sec. II it is shown that the Lagrangian model of the multibody structure is feedback equivalent to a particular form, here called *linear analytic* and *driftless under integral control.* Section III briefly addresses the design of the mathematical model used in Sec. IV, for control purposes. The continuous component and the piecewise constant part of the control law are computed in Secs. IV.A and IV.B, respectively, whereas the whole expression of the piecewise continuous feedback is given in Sec. IV.C. Section V is devoted to discuss some simulation results briefly.

II. On Feedback Equivalence of Nonholonomic Mechanical Dynamics

In this section the relationship between the kinematic equations and the dynamic models is investigated. The dynamic model will be proven to be equivalent to a suitable extension of the kinematic equation. Although the final result here obtained is substantially the same as in Ref. 18, it is here achieved showing the full transformation instead of using a projection onto a particular subset of the state space. Such an equivalence is at the basis of the design procedure proposed in Sec. IV; moreover, it is useful when generic underactuated structures with external inputs are considered.

The class of the nonholonomic constraints here considered is described by the expression

$$\omega(\xi)\dot{\xi} = 0 \tag{1}$$

where ξ is the n-dimensional vector of generalized coordinates and $\omega(\cdot)$ is a $(k \times n)$ matrix whose entries are real valued analytic functions. Equation (1) models the fact that at each point ξ there are k directions along which no evolution is allowed because of physical constraints, as nonskidding wheels or angular momentum conservation. In other words, at each configuration ξ infinitesimal displacements in only $(n-k)$ directions are possible. This local restriction, however, is not reflected in a reduction of the reachable configurations. A well-known example of such a behavior is the motion of a car: it cannot move sideways, but any point can be reached after a proper maneuver.

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From Eq. (1) the so-called kinematic model can be computed:

$$\dot{\xi} = \sum_{i=1}^{n-k} K_i(\xi)v_i = K(\xi)u \tag{2}$$

where $K(\xi)$ is a $n \times (n-k)$ matrix whose columns $K_i(\xi)$ are a basis for $\omega(\xi)^\perp$ [i.e., $\omega(\xi)K(\xi) = 0$ for all $\xi \in \mathbb{R}^n$]. Then Eq. (2) simply defines the admissible directions as any linear combination, with coefficients u_i, of the $K_i(\xi)$s.

Nonholonomic kinematic models can be used to check the admissibility of a given trajectory $\xi(t)$, $t \in [t_0, t_f]$: it must satisfy Eq. (2) for certain values $u_1, \ldots, u_{n-k}$. Moreover, it is possible to generate admissible paths by integrating Eq. (2) once $u(t)$ is chosen, or compute $u(t)$ in order to generate a path connecting two prefixed positions ξ^i and ξ^f (motion planning problem). Examples of such applications can be found in Refs. 7, 8, and 15.

From the practical point of view, the implementation of a controller designed on the kinematic model requires an additional regulator generating the needed torques.

With the purpose of getting a direct design procedure, a straightforward idea brings about the modification of the kinematic model [Eq. (1)] by adding integrators on the input variables, thus getting

$$\dot{\xi} = K(\xi)u, \qquad \dot{u} = a \tag{3}$$

Equation (3), denoted LADIC (linear analytic and driftless under integral control), defines a model with acceleration inputs in suitable coordinates, which must be expressed in terms of the actual control inputs.

To this purpose the LADIC form can always be achieved under feedback and coordinates transformations starting from a Lagrangian model subject to nonholonomic constraints, as shown next.

Starting from the dynamics of a mechanical system derived using a Euler–Lagrange approach in the generalized coordinates θ,

$$\ddot{\theta} = -J^{-1}(\theta)[F(\theta, \dot{\theta}) + G(\theta) - \omega^T(\theta)\lambda - B(\theta, \dot{\theta})\tau] \tag{4}$$

it is possible to compute, from the state space representation of Eq. (4) in the variables $(\theta, \dot{\theta})^T$, a change of coordinates and a static-state feedback, which transform Eq. (4) into Eq. (3) and vice versa.

Theorem 1: Nonholonomic dynamics in the form Eqs. (3) and (4) are feedback equivalent.

Proof: Consider the m-dimensional distribution $D = [\omega(\theta)J^{-1}(\theta)]^\perp$; let $\{\gamma_1(\theta), \ldots, \gamma_m(\theta)\}$, $m = (n-k)$, be a basis of D, i.e., $D = \operatorname{span}\{\gamma_1(\theta), \ldots, \gamma_m(\theta)\}$. Then, we can define the $n \times n$ matrix

$$H(\theta) = \begin{pmatrix} \gamma_1^T(\theta) \\ \vdots \\ \gamma_m^T(\theta) \\ \omega_1(\theta) \\ \vdots \\ \omega_k(\theta) \end{pmatrix} = \begin{pmatrix} \gamma^T(\theta) \\ \omega(\theta) \end{pmatrix} \tag{5}$$

We will show that the coordinates change:

$$\xi = \theta, \qquad \begin{pmatrix} u \\ \bar{u} \end{pmatrix} = H(\theta)\dot{\theta} \tag{6}$$

with $u \in \mathbb{R}^m$, $\bar{u} \in \mathbb{R}^k$, and the state feedback

$$\tau = c^{-1}(\xi, u)[a - b(\xi, u)] \tag{7}$$

with

$$b(\xi, u) = \dot{\gamma}^T(\xi)K(\xi)u - \gamma^T(\xi)J^{-1}(\xi)F[\xi, K(\xi)u]$$
$$- \gamma^T(\xi)J^{-1}(\xi)G(\xi)$$
$$c(\xi, u) = \gamma^T(\xi)J^{-1}(\xi)B[\xi, K(\xi)u]$$

for a suitable choice of $K(\xi)$, transform Eq. (4) into Eq. (3).

To prove that Eq. (6) is a change of coordinates, it is sufficient to prove the nonsingularity of $H(\theta)$ for all $\theta \in \mathbb{R}^n$. For this aim note that singularity occurs if and only if there exists at least one row of $\omega(\theta)$, say $\omega_i(\theta)$, such that $\omega_i^T(\theta) \in D$, i.e.,

$$\omega_j(\theta)J^{-1}(\theta)\omega_i^T(\theta) = 0, \qquad \forall j \in [1, k]$$

But $J(\theta)$ is the inertia matrix and thus positive definite; hence, $\omega_i(\theta)J^{-1}(\theta)\omega_i^T(\theta)$ is always different from 0 for $\omega_i(\theta) \neq 0$, and the nonsingularity of $H(\theta)$ follows.

The existence and invertibility of Eq. (7) depend on the nonsingularity of the matrix $c(\xi, u)$. This property holds if the system has as many actuators as admissible degrees of freedom, because full actuation implies that all of the matrices involved are full rank.

The inverse change of coordinates is then given by

$$\theta = \xi, \qquad \dot{\theta} = H^{-1}(\theta)|_{\theta=\xi}\begin{pmatrix} u \\ \bar{u} \end{pmatrix} = H^{-1}(\xi)\begin{pmatrix} u \\ \bar{u} \end{pmatrix}$$

and because $\bar{u} = 0$ by construction, denoting by $K(\xi)$ the first $m = n - k$ columns of $H^{-1}(\xi)$, it becomes

$$\theta = \xi, \qquad \dot{\theta} = K(\xi)u$$

In the new state variables Eq. (4) takes the form

$$\dot{\xi} = \dot{\theta} = K(\xi)u$$
$$\dot{u} = [\dot{\gamma}^T(\theta)\dot{\theta} + \gamma^T(\theta)\ddot{\theta}]_{(\theta,\dot{\theta})^T = [\xi, K(\xi)u]^T}$$
$$= [\dot{\gamma}^T(\theta)\dot{\theta} - \gamma^T(\theta)J^{-1}(\theta)F(\theta, \dot{\theta}) - \gamma^T(\theta)J^{-1}(\theta)G(\theta)$$
$$+ \gamma^T(\theta)J^{-1}(\theta)B(\theta, \dot{\theta})\tau]_{(\theta,\dot{\theta})^T = [\xi, K(\xi)u]^T}$$
$$= \dot{\gamma}^T(\xi)K(\xi)u - \gamma^T(\xi)J^{-1}(\xi)F[\xi, K(\xi)u]$$
$$- \gamma^T(\xi)J^{-1}(\xi)G(\xi) + \gamma^T(\xi)J^{-1}(\xi)B[\xi, K(\xi)u]\tau$$
$$= b(\xi, u) + c(\xi, u)\tau$$

which, under the feedback [Eq. (7)], reduces to Eq. (3).

To end the proof, we have to show that the subsystem $\dot{\xi} = K(\xi)u$ actually represents the kinematic model. Starting from the definition of $H(\xi)$, one has

$$H(\xi)H^{-1}(\xi) = \begin{pmatrix} \gamma^T(\xi) \\ \omega(\xi) \end{pmatrix}(K(\xi) \quad *)$$
$$= \begin{pmatrix} \gamma^T(\xi)K(\xi) & * \\ \omega(\xi)K(\xi) & * \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$$

from which $\omega(\xi)K(\xi) = 0$ follows, according to the already introduced definition of kinematic model. □

Remark 1: Equation (3) is not unique, i.e., there exist different choices of state variables and related feedback laws that give the same structure. In fact the proof of Theorem 1 holds also if $\gamma^T(\xi)$ is replaced by $Q(\xi)\gamma^T(\xi)$, for any nonsingular $(m \times m)$ matrix $Q(\xi)$. Equivalently, Eq. (3) is invariant under coordinates change of the form

$$\xi' = T\xi, \qquad u' = Q(\xi)u$$

and feedback

$$a = Q^{-1}(T^{-1}\xi')a' + \dot{Q}^{-1}(T^{-1}\xi')u'$$

for any nonsingular matrices $T(n \times n)$ and $Q(\xi)$ $(m \times m)$.

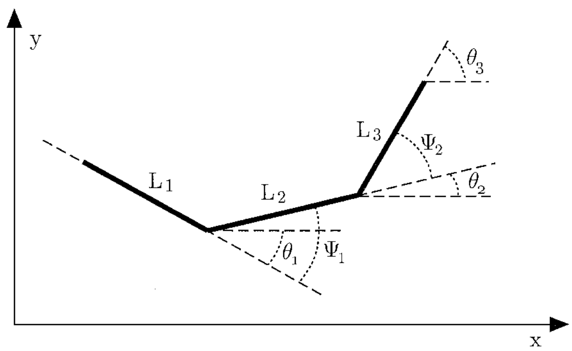

Fig. 1 Mechanical structure for $N = 3$.

III. Planar Three-Link Mechanical Structure

Consider the planar structure composed by N rigid bodies interconnected by frictionless revolution joints, defining an open kinematic chain. Such a structure, for $N = 3$, is depicted in Fig. 1. Assuming as control inputs the torques at the joints, the conservation of both the momentum and the angular momentum follows. Both of them can be assumed, without loss of generality, equal to zero. The mathematical model is computed in the sequel according to Refs. 10 and 11.

In a Lagrangian formulation the absolute angles θ_i of the N bodies define a set of generalized coordinates, and the vector $\theta = (\theta_1 \cdots \theta_N)^T$ completely describes the instantaneous configuration of the structure.

For the present case the potential energy is zero, and the Lagrangian takes the form

$$L(\theta, \dot\theta) = \tfrac{1}{2}\dot\theta^T J(\theta)\dot\theta$$

where $J(\theta)$ is the inertia matrix, whose entries are[11]

$$J_{i,j}(\theta) = \begin{cases} I_i & \text{if} \quad i = j \\ h^1_{i,j}\cos(\theta_j - \theta_i) + h^2_{i,j}\sin(\theta_j - \theta_i) & \text{if} \quad i \neq j \end{cases}$$

and coefficients $h^1_{\cdot,\cdot}\ h^2_{\cdot,\cdot}$ such that $J_{i,j}(\theta) = J_{j,i}(\theta)$.

Let $\psi_i = \theta_{i+1} - \theta_i$, $i = 1, \ldots, N-1$, be the relative angles between the link L_i and L_{i+1} and denote by ψ the corresponding $(n-1)$-dimensional vector.

Defining the $(N-1) \times N$ full rank matrix P by

$$P_{i,j} = \begin{cases} -1 & \text{if} \quad i = j \\ +1 & \text{if} \quad i = j - 1 \\ 0 & \text{otherwise} \end{cases}$$

the relationship between θ and ψ can be written as

$$\psi = P\theta$$

Denoting, finally, by $\tau = (\tau_1 \cdots \tau_{N-1})^T$ the joint torques, the motion equations for the unconstrained system become

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial L(\theta, \dot\theta)}{\partial \dot\theta}\right]^T - \left[\frac{\partial L(\theta, \dot\theta)}{\partial \theta}\right]^T = P^T\tau$$

which can be rewritten as

$$J(\theta)\ddot\theta + F(\theta, \dot\theta) = P^T\tau \tag{8}$$

where $P^T\tau$ represents the vector of the generalized torques.

The constraint expressing the conservation of the total angular momentum must be taken into account. It takes the form:

$$\mathbf{1}^T J(\theta)\dot\theta = \omega(\theta)\dot\theta = \text{const} = 0 \tag{9}$$

where $\mathbf{1}^T = (1\ \ 1\ \ \cdots\ \ 1)$ is the unit n-dimensional row vector. Equation (9) can be considered, for its form, as a classical nonholonomic constraint.[5,19]

Finally, in our case the mathematical model takes the form Eq. (4), with

$$G(\theta) = 0, \qquad B(\theta, \dot\theta) = P^T, \qquad \lambda = 0$$

In particular, the condition $\lambda = 0$ means that we are not dealing with a real nonholonomic system, but with an underactuated one that can be assimilated to a nonholonomic system by means of the constraint (9). That is the reason why in this context the design procedure proposed here can make use of some tools introduced in the framework of nonholonomic mobile robotics.

We will refer to the three-links open kinematic chain depicted in Fig. 1, whose mathematical model is given by Eq. (8) with $N = 3$. In the state-space representation it takes the form

$$\begin{pmatrix} \dot\theta \\ \ddot\theta \end{pmatrix} = \begin{pmatrix} \dot\theta \\ -J^{-1}(\theta)[F(\theta, \dot\theta) - P^T\tau] \end{pmatrix}$$

We will apply Theorem 1, with H in Eq. (5), given by

$$H(\theta) = \begin{pmatrix} P \\ \mathbf{1}^T J(\theta) \end{pmatrix}$$

because $PJ^{-1}(\theta)J(\theta)\mathbf{1} = P\mathbf{1} = 0$. Then one obtains

$$\begin{pmatrix} \xi \\ u \\ \bar u \end{pmatrix} = \begin{pmatrix} \theta \\ H(\theta)\dot\theta \end{pmatrix} = \begin{pmatrix} \theta \\ P\dot\theta \\ \mathbf{1}^T J(\theta)\dot\theta \end{pmatrix} = \begin{pmatrix} \theta \\ P\dot\theta \\ 0 \end{pmatrix} \tag{10}$$

with $\xi \in \mathbb{R}^3$, $u \in \mathbb{R}^2$, and $\bar u \in \mathbb{R}$.

The inverse transformation is given by

$$\theta = \xi, \qquad \dot\theta = H^{-1}(\xi)\begin{pmatrix} u \\ \bar u \end{pmatrix} = K(\xi)u$$

where $K(\xi)$ is composed by the first two columns of $H^{-1}(\theta)|_{\theta=\xi}$.

After the coordinates change the system becomes

$$\dot\xi = \dot\theta|_{\dot\theta = K(\xi)u} = K(\xi)u$$

$$\dot u = P\ddot\theta|_{(\theta,\dot\theta)^T = [\xi, K(\xi)u]^T} = -PJ^{-1}(\xi)F[\xi, K(\xi)u]$$

$$+ PJ^{-1}(\xi)P^T\tau = b(\xi, u) + c(\xi)\tau$$

where $c(\xi)$ is a nonsingular (2×2) matrix. The structure as in Eq. (3) is then obtained under the feedback

$$\tau = c^{-1}(\xi)[a - b(\xi, u)]$$

$$= [PJ^{-1}(\xi)P^T]^{-1}\{a + PJ^{-1}(\xi)F[\xi, K(\xi)u]\} \tag{11}$$

The numerical data assumed for the mechanical structure under study are the ones in Ref. 8, noted in Table 1.

Then the inertia matrix is

$$J(\theta) = \begin{pmatrix} 4 & 3\cos(\theta_2 - \theta_1) & \cos(\theta_3 - \theta_1) \\ 3\cos(\theta_2 - \theta_1) & 8 & 3\cos(\theta_3 - \theta_2) \\ \cos(\theta_3 - \theta_1) & 3\cos(\theta_3 - \theta_2) & 4 \end{pmatrix}$$

and, by computations, one has

Table 1 Numerical data for the inertia matrix of the structure

I	h^1	1	2	3	h^2	1	2	3
$I_1 = 4$	1	——	3	1	1	——	0	0
$I_2 = 8$	2	3	——	3	2	0	——	0
$I_3 = 4$	3	1	3	——	3	0	0	——

$$J^{-1}(\theta) = \frac{1}{\Delta}\begin{pmatrix} 32-9\cos^2(\theta_{23}) & 3\cos(\theta_{13})\cos(\theta_{23})-12\cos(\theta_{12}) & 9\cos(\theta_{12})\cos(\theta_{23})-8\cos(\theta_{13}) \\ * & 16-\cos^2(\theta_{13}) & 3\cos(\theta_{12})\cos(\theta_{13})-12\cos(\theta_{23}) \\ * & * & 32-9\cos^2(\theta_{12}) \end{pmatrix}$$

where

$$\theta_{12} = \theta_2 - \theta_1 \qquad \theta_{13} = \theta_3 - \theta_1 \qquad \theta_{23} = \theta_3 - \theta_2$$

$$\Delta = 128 - 36\cos^2(\theta_2-\theta_1) - 36\cos^2(\theta_3-\theta_2) - 8\cos^2(\theta_3-\theta_1) + 18\cos(\theta_2-\theta_1)\cos(\theta_3-\theta_1)\cos(\theta_3-\theta_2)$$

and

$$F(\theta,\dot\theta) = \frac{\mathrm{d}J(\theta)}{\mathrm{d}t}\dot\theta - \frac{1}{2}\left(\frac{\partial}{\partial\theta}[\dot\theta^T J(\theta)\dot\theta]\right)^T = \begin{pmatrix} -3\sin(\theta_2-\theta_1)\dot\theta_2^2 - \sin(\theta_3-\theta_1)\dot\theta_3^2 \\ 3\sin(\theta_2-\theta_1)\dot\theta_1^2 - 3\sin(\theta_3-\theta_2)\dot\theta_3^2 \\ \sin(\theta_3-\theta_1)\dot\theta_1^2 + 3\sin(\theta_3-\theta_2)\dot\theta_2^2 \end{pmatrix}$$

Computing the coordinates change [Eq. (10)] and the state feedback [Eq. (11)], the LADIC structure Eq. (3) is achieved, with

$$K(\xi) = \begin{pmatrix} [-12-3c_{12}(\xi)-c_{13}(\xi)-6c_{23}(\xi)]/d(\xi) & [-4-c_{13}(\xi)-3c_{23}(\xi)]/d(\xi) \\ [4+3c_{12}(\xi)+c_{13}(\xi)]/d(\xi) & [-4-c_{13}(\xi)-3c_{23}(\xi)]/d(\xi) \\ [4+3c_{12}(\xi)+c_{13}(\xi)]/d(\xi) & [12+6c_{12}(\xi)+c_{1,3}(\xi)+3c_{23}(\xi)]/d(\xi) \end{pmatrix}$$

where

$$d(\xi) = 16 + 6c_{12}(\xi) + 2c_{13}(\xi) + 6c_{23}(\xi)$$

$$c_{12}(\xi) = \cos(\xi_1 - \xi_1), \qquad c_{13}(\xi) = \cos(\xi_3 - \xi_1)$$

$$c_{23}(\xi) = \cos(\xi_3 - \xi_2)$$

As intuition suggests, $K(\xi)$ does not depend on the values of ξ_1, ξ_2, and ξ_3 separately, but only on the relative angles $\psi_1 = (\xi_2 - \xi_1) = (\theta_2 - \theta_1)$ and $\psi_2 = (\xi_3 - \xi_2) = (\theta_3 - \theta_2)$ already introduced.

Thus a new state-space transformation can be performed, taking as new state variables ψ_1, ψ_2, and one of the ξ_i. If ξ_2 is chosen, i.e., the central body orientation, defining the new coordinates

$$x = \begin{pmatrix}\psi_1\\ \psi_2\\ \xi_2\end{pmatrix} = \begin{pmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 0 & 1 & 0\end{pmatrix}\xi = T\xi \tag{12}$$

while u is left unchanged, one obtains

$$\dot x = \bar K(x)u \tag{13a}$$

$$\dot u = a \tag{13b}$$

where

$$\bar K(x) = TK(\xi)|_{\xi = T^{-1}x} = \begin{pmatrix}1 & 0\\ 0 & 1\\ s_1(x) & s_2(x)\end{pmatrix} = (\gamma_1(x) \quad \gamma_2(x)) \tag{14}$$

with

$$s_1(x) = \frac{4 + 3\cos(x_1) + \cos(x_1+x_2)}{16 + 6\cos(x_1) + 6\cos(x_2) + 2\cos(x_1+x_2)}$$

$$s_2(x) = -\frac{4 + 3\cos(x_2) + \cos(x_1+x_2)}{16 + 6\cos(x_1) + 6\cos(x_2) + 2\cos(x_1+x_2)} \tag{15}$$

and

$$\gamma_1(x) = \begin{pmatrix}1\\ 0\\ s_1(x)\end{pmatrix}, \qquad \gamma_2(x) = \begin{pmatrix}0\\ 1\\ s_2(x)\end{pmatrix} \tag{16}$$

without loosing the LADIC structure, according to remark 1 with $Q(\xi) = I$ and T as in Eq. (12).

IV. Piecewise Continuous Control Law for Attitude and Configuration Control

The present section deals with the design of the controller for achieving the desired attitude and configuration. After a preliminary study of the solvability of the problem and some remarks on the computations and the properties of the sampled model, the computation of the control law is developed in Secs. IV.A and IV.B. More precisely, in IV.A it will be shown that the LADIC model (11) can be transformed, under feedback and coordinates change, into a computable sampled dynamics. Such a feedback and coordinates transformation constitute the inner loop of the control scheme here proposed and depicted in Fig. 2. On the basis of the sampled model, an inversion technique is applied in Sec. IV.B to compute the control law for achieving the desired orientation and configuration.

The existence of a solution to our control problem results from the full accessibility of the dynamic model (13a, 13b), i.e., the existence of a control law that brings the state from any initial value to any final one. At this end we recall from Ref. 20 that the kinematic model (2), represented in the present case by (13a), is fully accessible if the Lie algebra generated by $\gamma_1(x)$ and $\gamma_2(x)$ in Eq. (14) has dimension three at each point x of $\mathbb{R}^3$. Recalling that the Lie product of two vector fields τ_1 and τ_2 in local coordinates is $[\tau_1, \tau_2](x) = (\partial\tau_2/\partial x)\tau_1(x) - (\partial\tau_1/\partial x)\tau_2(x)$, the computation of $[\gamma_1, \gamma_2](x)$ and $[\gamma_1, [\gamma_1, \gamma_2]](x)$ can be easily performed. After computations, two vector fields are obtained, with only the last component different from zero, and are given by

$$n_1(x)/2[8 + 3\cos(x_1) + 3\cos(x_2) + \cos(x_1+x_2)]^2$$

with

$$n_1(x) = -12\sin(x_1) - 9\cos(x_2)\sin(x_1) - 3\cos(x_1+x_2) - 12\sin(x_2) - 9\cos(x_1)\sin(x_2) + 8\sin(x_1+x_2) - 3\cos(x_1+x_2)\sin(x_2) + 3\cos(x_1)\sin(x_1+x_2) + 3\cos(x_2)\sin(x_1+x_2)$$

for $[\gamma_1, \gamma_2](x)$, and

$$n_2(x)/4[8 + 3\cos(x_1) + 3\cos(x_2) + \cos(x_1+x_2)]^3$$

with

$$n_2(x) = -84 - 165\cos(x_1) + 27\cos(2x_1) - 81\cos(x_1-x_2) - 36\cos(x_2) + 11\cos(x_1+x_2) + \cos[2(x_1+x_2)] + 12\cos(2x_1+x_2) + 15\cos(x_1+2x_2)$$

for $[\gamma_1, [\gamma_1, \gamma_2]](x)$.

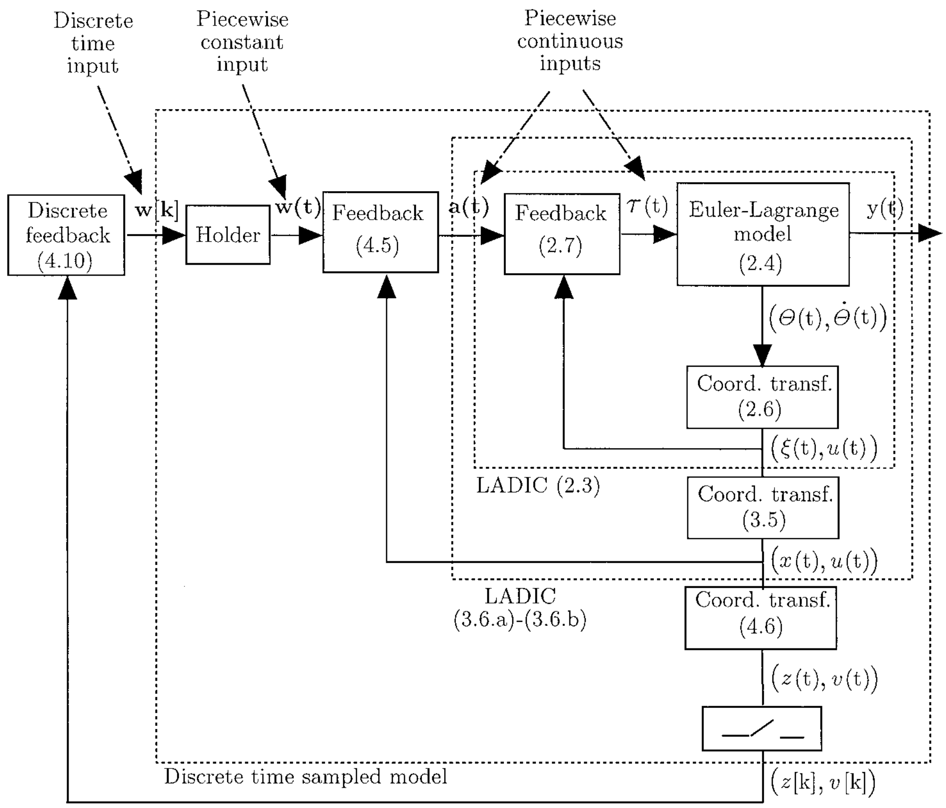

Fig. 2 Control scheme.

On this basis it is not difficult to verify that the dimension of

$$(\gamma_1, \gamma_2, [\gamma_1, \gamma_2], [\gamma_1, [\gamma_1, \gamma_2]])(x)$$

is three everywhere, so that Eq. (13a) is fully accessible. Full accessibility of the dynamic model (13a) and (13b) follows Ref. 20, and because of the use of the invertible feedback [Eq. (11)], the full accessibility of Eqs. (13a) and (13b) follows too.

In conclusion there exists a control law that brings the state from any initial value to any final one, i.e., the problem of computing a control law for a reorientation and reconfiguration maneuver of our articulated structure is always solvable.

As announced at the beginning of the section, before starting with the design procedure some recalls on sampling are in order. Given a nonlinear dynamics of the form

$$\dot{x} = f(x) + \sum_{i=1}^{m} u_i g_i(x) \tag{17}$$

with $x \in M \subset \mathbb{R}^n$, $u \in U \subset \mathbb{R}^m$, $f, g_1, \ldots, g_m$ are real analytic vector fields on $M \subset \mathbb{R}^n$. Assuming piecewise constant inputs over the time intervals $[k\delta, (k+1)\delta[$, the solution $x[k+1]$ at time $t = (k+1)\delta$, starting from $x[k]$ at time $t = k\delta$, at least for small values of δ is an analytic function $F_\delta : M \times U \to M$, which defines the equivalent sampled model of Eq. (17) (Ref. 21):

$$x[k+1] = F_\delta(x[k], u[k]) \tag{18}$$

Definitions[16]*:* If the function F_δ is defined for any positive δ, then it will be called the *sampled closed form* of Eq. (17) or its *equivalent exact sampled model.* A particular case occurs when F_δ is a polynomial in δ of finite order $\bar{k}$. In this case it will be called the *finite equivalent sampled model* of order $\bar{k}$ of Eq. (17).

According to Ref. 16, as can easily be verified, if Eq. (17) exhibits a polynomial subtriangular structure [i.e., each component $f_i(x, u)$ is polynomial in its arguments, and under possible reordering, $f_i(x, u)$ does not depend on x_j, $j = i, \ldots, n$], then it admits a finite equivalent sampled model and vice versa.

The finite discretizability property can be extended, considering a state feedback and a coordinates change, according to the following definition.

Definition: A dynamics [Eq. (17)] is said to be finitely feedback discretizable if there exist a state feedback and a coordinates change under which the system exhibits a polynomial subtriangular structure.

A. Continuous Inner Feedback

Let us preliminarily consider a dynamics of the form

$$\dot{x} = f(x, u) \tag{19a}$$

$$\dot{u} = G(u)a \tag{19b}$$

with input $a \in \mathbb{R}^m$.

Lemma 1: If Eq. (19a) is finitely discretizable under feedback $u = \beta(x)v$, the same property holds for the extended system Eqs. (19a) and (19b).

Proof: If a feedback $u = \beta(x)v$ and a coordinates change $z = \Phi(x)$ transform $\dot{x} = f(x, u)$ into

$$\dot{z} = \left\{ \frac{\partial \Phi(x)}{\partial x} f[x, \beta(x)v] \right\}_{x = \Phi^{-1}(z)} = D(z, v)$$

with $D(z, v)$ polynomial subtriangular, then the feedback

$$a = [\beta^{-1}(x)G(u)]^{-1}[w - \dot{\beta}^{-1}(x)u]$$

and the coordinates change

$$\begin{pmatrix} z \\ v \end{pmatrix} = \begin{pmatrix} \Phi(x) \\ \beta^{-1}(x)u \end{pmatrix} \tag{20}$$

applied to Eqs. (19a) and (19b) give

$$\begin{pmatrix} \dot{z} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial \Phi(x)}{\partial x} f(x, u) \\ \dot{\beta}^{-1}(x)u + \beta^{-1}(x)G(u)a \end{pmatrix}_{(z,v)} = \begin{pmatrix} D(z, v) \\ w \end{pmatrix}$$

which is polynomial subtriangular. Indeed, the polynomial form is preserved, and, after reordering, from

$$\begin{pmatrix} \dot{v} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} w \\ D(z, v) \end{pmatrix}$$

it is easily understood that subtriangularity is maintained too. The finite discretization property follows. □

From Lemma 1 it immediately follows that finite discretizability of Eq. (9) implies the finite discretizability of Eq. (10). Finally, because of Theorem 1, Proposition 1 follows:

Proposition 1: The finite feedback discretizability of Eq. (9) implies the finite feedback discretizability of Eq. (11).

With this in mind let us go back to our case study and consider the kinematic model in Eq. (13a). It is a matter of computation to verify that it takes a polynomial subtriangular form under the following feedback:

$$u = \beta(x)v = \begin{pmatrix} 1 & 0 \\ -\dfrac{L^2_{\gamma_1(x)}\lambda(x)}{L_{\gamma_2(x)}L_{\gamma_1(x)}\lambda(x)} & \dfrac{1}{L_{\gamma_2(x)}L_{\gamma_1(x)}\lambda(x)} \end{pmatrix} v$$

and coordinates transformation

$$z = Z(x) = \begin{pmatrix} x_1 \\ L_{\gamma_1(x)}\lambda(x) \\ \lambda(x) \end{pmatrix}$$

where

$$\lambda(x) = \frac{\left\{\sqrt{6}[16 - 12\cos(x_1)]m(x_1, x_2) + (3\pi - 6x_2 - 12x_3)r(x_1)\right\}[-8 - 3\cos(x_1) + \sin(x_1)]^2}{6r(x_1)[-3 - 4\cos(x_1) + 12\sin(x_1)]}$$

with

$$r(x_1) = \sqrt{39 + 28\cos(x_1) + 3\cos(2x_1)}$$

and

$$m(x_1, x_2) = \arctan\frac{-5\sqrt{2} - \cos(\pi/4 - x_1) + 3\cos(3\pi/4 - x_1)}{\sqrt{3}r(x_1)} - \arctan\frac{\sec(x_2/2)[3\sin(x_1 - x_2/2) - \sin(x_1 + x_2/2) - 10\sin(x_2/2)]}{\sqrt{6}r(x_1)}$$

One has

$$\dot{z}_1 = v_1, \qquad \dot{z}_2 = v_2, \qquad \dot{z}_3 = z_2 v_1$$

According to Lemma 1, the state feedback

$$\begin{aligned} a &= \dot{\beta}(x)\beta^{-1}(x)u + \beta(x)w \\ &= \begin{pmatrix} 0 & 0 \\ -\dfrac{\mathrm{d}}{\mathrm{d}t}\dfrac{L^2_{\gamma_1(x)}\lambda(x)}{L_{\gamma_2(x)}L_{\gamma_1(x)}\lambda(x)} & \dfrac{\mathrm{d}}{\mathrm{d}t}\dfrac{1}{L_{\gamma_2(x)}L_{\gamma_1(x)}\lambda(x)} \end{pmatrix} \\ &\quad \times \begin{pmatrix} 1 & 0 \\ L^2_{\gamma_1(x)}\lambda(x) & L_{\gamma_2(x)}L_{\gamma_1(x)}\lambda(x) \end{pmatrix} u \\ &\quad + \begin{pmatrix} 1 & 0 \\ -\dfrac{L^2_{\gamma_1(x)}\lambda(x)}{L_{\gamma_2(x)}L_{\gamma_1(x)}\lambda(x)} & \dfrac{1}{L_{\gamma_2(x)}L_{\gamma_1(x)}\lambda(x)} \end{pmatrix} w \end{aligned} \tag{21}$$

and the coordinates change

$$\begin{pmatrix} z \\ v \end{pmatrix} = \begin{pmatrix} Z(x) \\ \beta^{-1}(x)u \end{pmatrix} \tag{22}$$

transform Eq. (16) into a polynomial subtriangular dynamics of the form

$$\dot{z}_1 = v_1, \qquad \dot{z}_2 = v_2, \qquad \dot{z}_3 = z_2 v_1, \qquad \dot{v}_1 = w_1, \qquad \dot{v}_2 = w_2 \tag{23}$$

Equation (23) admits a finite sampled model, which is computed in the sequel.

B. Piecewise Constant Feedback

For finitely discretizable dynamics the problem of computing a control law for steering a given initial state x^i to a final one x^f in finite time T can be addressed as follows. Assume any input u_i piecewise constant over intervals of amplitude T/r_i, $i = 1, \ldots, m$ and compute the sampled dynamics

$$\begin{aligned} x[k+1] &= F_T(x[k], u_{1,1}[k], \ldots, u_{1,r_1}[k], \ldots, u_{m,1}[k], \ldots, u_{m,r_m}[k]) \\ &= F_T(x[k], \tilde{u}[k]) \end{aligned} \tag{24}$$

Equation (24) defines the so-called *multirate sampled model*,[22] which is finitely computable if the usual sampled one is.

In Eq. (24) $x[k]$ and $x[k+1]$ are linked by an increased number of independent variables. From any given initial state $x[k] = x^i$, any state x^f belonging to the image of $F_T(x^i, \cdot)$ can be attained in T instants of time, i.e., $x^f = x[k+1]$. If in particular the choice of the overdimensioned rate on the input channels gets Eq. (24) invertible at x^i, one uniquely computes

$$\tilde{u}[k] = F_T^{-1}(x^i, x^f)$$

With reference to our case study, starting from Eq. (23) the finite sampled dynamics (18) is given by

$$\begin{aligned} z_1[k+1] &= z_1[k] + \delta v_1[k] + (\delta^2/2)w_1[k] \\ z_2[k+1] &= z_2[k] + \delta v_2[k] + (\delta^2/2)w_1[k] \\ z_3[k+1] &= z_3[k] + \delta z_2[k]v_1[k] + (\delta^2/2)(v_1[k]v_2[k] + z_2[k]w_1[k]) \\ &\quad + (\delta^3/3!)(2w_1[k]v_2[k] + v_1[k]w_2[k]) + (\delta^4/4!)3w_1[k]w_2[k] \\ v_1[k+1] &= v_1[k] + \delta w_1[k] \\ v_2[k+1] &= v_2[k] + \delta w_2[k] \end{aligned}$$

With the choice of $r_1 = 2$ and $r_2 = 3$, an invertible map $F_T(x^i, \cdot)$ is obtained with the following assumptions on the control variables:

$$w_1[k] = \begin{cases} w_{1,1} & \text{if} \quad 0 \le t < 2\delta \\ w_{1,2} & \text{if} \quad 2\delta \le t < 4\delta \end{cases}$$

$$w_2[k] = \begin{cases} w_{2,1} & \text{if} \quad 0 \le t < \delta \\ w_{2,2} & \text{if} \quad \delta \le t < 3\delta \\ w_{2,3} & \text{if} \quad 3\delta \le t < 4\delta \end{cases} \tag{25}$$

with $\delta = T/4$ to be fixed according to the planned time duration of the maneuver.

In this case the expressions for the controls $(w_{i,j})$ are obtained by inversion of the multirate sampled dynamics

$$z_1[k+1] = z_1[k] + 4\delta v_1[k] + 6\delta^2 w_{1,1} + 2\delta^2 w_{1,2}$$

$$z_2[k+1] = z_2[k] + 4\delta v_2[k] + (\delta^2/2)(7w_{2,1} + 8w_{2,2} + w_{2,3})$$

$$\begin{aligned} z_3[k+1] = {} & z_3[k] + 4\delta z_2[k]v_1[k] + 2\delta^2(4v_1[k]v_2[k] \\ & + 3z_2[k]w_{1,1} + z_2[k]w_{1,2}) + (\delta^3/6)(88v_2[k]w_{1,1} \\ & + 40v_2[k]w_{1,2} + 37v_1[k]w_{2,1} + 26v_1[k]w_{2,2} + v_1[k]w_{2,3}) \\ & + (\delta^4/24)(281w_{1,1}w_{2,1} + 136w_{1,2}w_{2,1} + 207w_{1,1}w_{2,2} \\ & + 129w_{1,2}w_{2,2} + 8w_{1,1}w_{2,3} + 7w_{1,2}w_{2,3}) \end{aligned}$$

$$v_1[k+1] = v_1[k] + 2\delta w_{1,1} + 2\delta w_{1,2}$$

$$v_2[k+1] = v_2[k] + \delta w_{2,1} + 2\delta w_{2,2} + \delta w_{2,3}$$

giving

$$w_{1,1} = \frac{z_1^f - z_1^i}{4\delta^2}, \qquad w_{1,2} = \frac{-z_1^f + z_1^i}{4\delta^2}, \qquad w_{2,1} = \frac{z_2^f - z_2^i}{3\delta^2} + \frac{-2\delta(4+73\delta)(z_1^f - z_1^i)(-z_2^f + z_2^i) + 288\delta^2(z_1^f z_2^i - z_1^i z_2^i - z_3^f + z_3^i)}{180\delta^4(-z_1^f + z_1^i)}$$

$$w_{2,2} = \frac{-\left[-2\delta(4+73\delta)(z_1^f - z_1^i)(-z_2^f + z_2^i) + 288\delta^2(z_1^f z_2^i - z_1^i z_2^i - z_3^f + z_3^i)\right]}{180\delta^4(-z_1^f + z_1^i)}$$

$$w_{2,3} = \frac{-z_2^f + z_2^i}{3\delta^2} + \frac{-2\delta(4+73\delta)(z_1^f - z_1^i)(-z_2^f + z_2^i) + 288\delta^2(z_1^f z_2^i - z_1^i z_2^i - z_3^f + z_3^i)}{180\delta^4(-z_1^f + z_1^i)} \tag{26}$$

once the initial and final values for the velocities v_i are fixed equal to zero (rest-to-rest motion).

Equation (25) can be used every time that $z_1^f - z_1^i \neq 0$ holds. Otherwise different multirate orders must be chosen to ensure the invertibility of the dynamics [Eq. (24)]. This is the case of the second simulation in Sec. V.

C. Complete Control Law

On the basis of these results, it is possible to compute the control input τ for system (3) as a feedback from the state variables $(\theta, \dot{\theta})$. In fact, according to all of the transformations performed, one has, from Eqs. (15) and (16),

$$\tau = [PJ^{-1}(\theta)P^T]^{-1}[a + PJ^{-1}(\theta)F(\theta, \dot{\theta})]$$

and for Eqs. (21) and (22),

$$\begin{aligned} \tau = {} & (PJ^{-1}(\theta)P^T)^{-1}[\dot{\beta}(T\theta)\beta^{-1}(T\theta)P\dot{\theta} + \beta(T\theta)w \\ & + PJ^{-1}(\theta)F(\theta, \dot{\theta})] \end{aligned} \tag{27}$$

Expression (27) defines the state feedback, which depends on the piecewise constant external input $w[k]$, for example, the one computed in Eqs. (26). Consequently, τ is a continuous function over each time interval $[0, \delta]$, $[\delta, 2\delta]$, $[2\delta, 3\delta]$, and $[3\delta, 4\delta]$.

The final control scheme, composed by an inner continuous time loop and a digital external one, is depicted in Fig. 2.

V. Simulation Results

Simulations have been carried out to show the effectiveness of the computed controller. To verify that the multiplier λ in Eq. (4) is identically equal to zero in the present case of a multilink structure, as discussed in Sec. III, it will be computed during the simulations. To do so, the following expression will be used: from Eq. (2), which holds during all the motion, one has

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{1}^T J(\theta)\dot{\theta} = 0$$

that is,

$$\mathbf{1}^T \dot{J}(\theta)\dot{\theta} + \mathbf{1}^T J(\theta)\ddot{\theta} = 0$$

Then, taking into account Eq. (3), one has

$$\mathbf{1}^T \dot{J}(\theta)\dot{\theta} + \mathbf{1}^T[-F(\theta, \dot{\theta}) + J(\theta)\mathbf{1}\lambda + P^T\tau] = 0$$

which gives

$$\lambda = [\mathbf{1}^T J(\theta)\mathbf{1}]^{-1}\mathbf{1}^T[-\dot{J}(\theta)\dot{\theta} + F(\theta, \dot{\theta})] \tag{28}$$

because $\mathbf{1}^T P^T = 0$.

The values of the parameters are in Table 1, whereas the initial and final positions are given by

$$\theta^i = \begin{pmatrix} 0.2 - \pi/2 \\ 0 \\ \pi/2 - 0.2 \end{pmatrix}, \qquad \dot{\theta}^i = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\theta^f = \begin{pmatrix} 0.2 - \pi/2 - \pi/4 \\ 0 \\ \pi/2 - 0.2 + \pi/4 \end{pmatrix}, \qquad \dot{\theta}^f = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

The results of the simulation performed are presented in the following figures. Figures 3 and 4 depict the evolutions of the angles

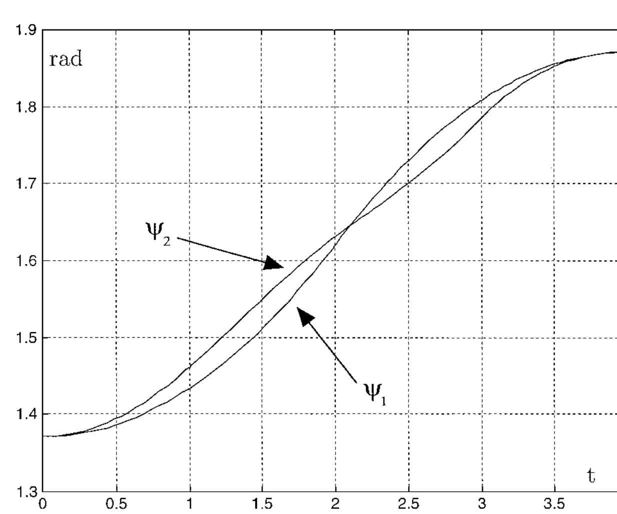

Fig. 3 Relative angles $\psi_1 = \theta_2 - \theta_1$ and $\psi_2 = \theta_3 - \theta_2$.

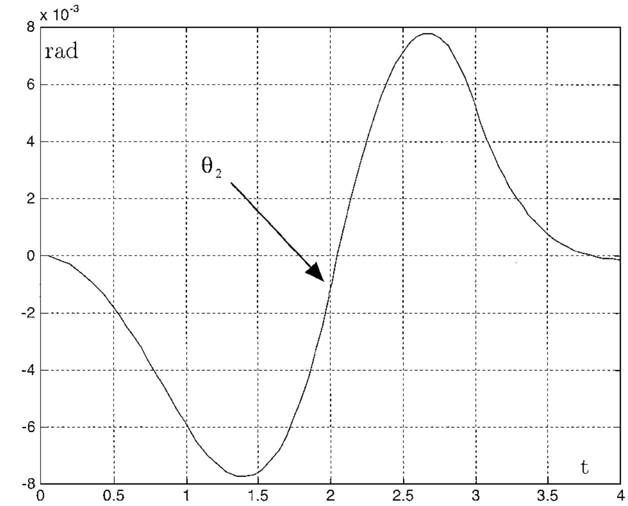

Fig. 4 Absolute angle θ_2.

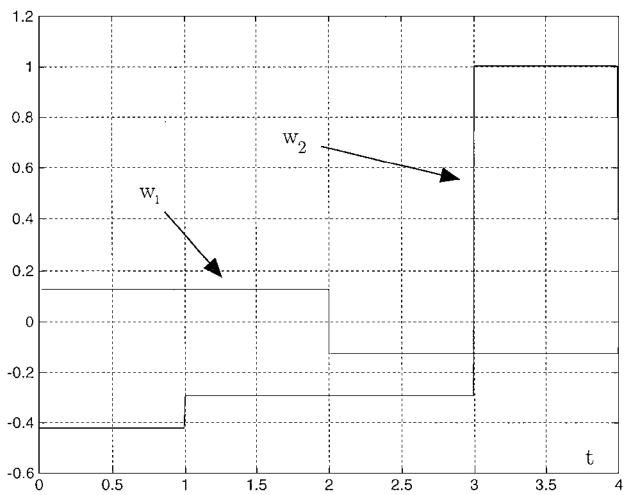

Fig. 5 Piecewise constant input w_1 and w_2.

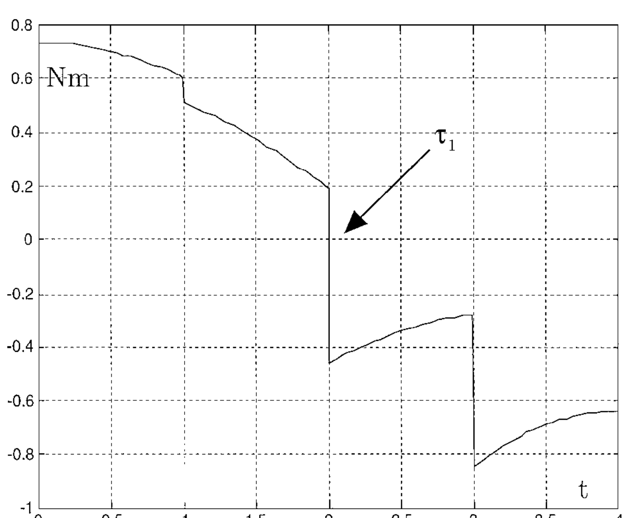

Fig. 6 Control torque τ_1.

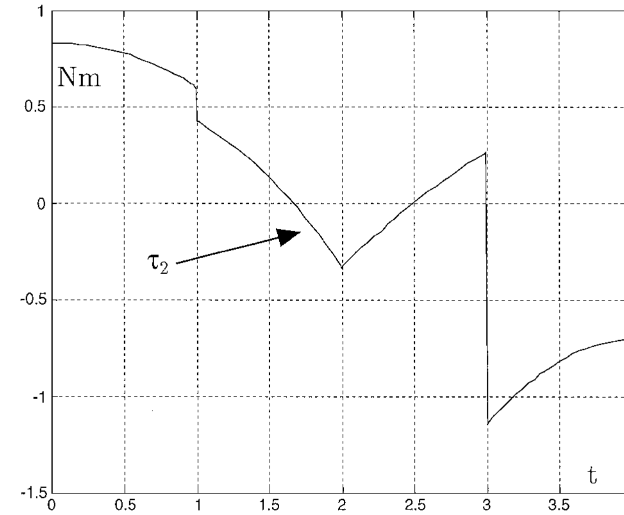

Fig. 7 Control torque τ_2.

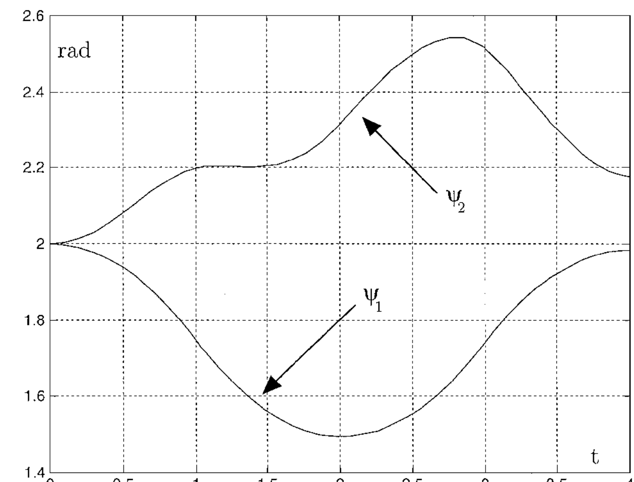

Fig. 8 Relative angles $\psi_1 = \theta_2 - \theta_1$ and $\psi_2 = \theta_3 - \theta_2$.

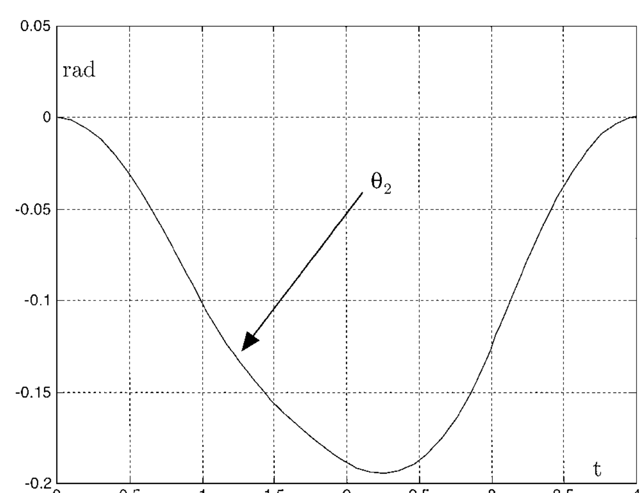

Fig. 9 Absolute angle θ_2.

ψ_1, ψ_2, and θ_2, showing that the desired maneuver is smoothly attained.

Figure 5 depicts the digital part of the control law.

Figures 6 and 7 depict the applied torques.

As pointed out in Sec. IV, the control law (22) is defined for $z_1^f - z_1^i \neq 0$. The second simulation, performed with the same parameters values and with the following initial and final conditions

$$\theta^i = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}, \quad \dot{\theta}^i = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad \theta^f = \begin{pmatrix} -2 \\ 0 \\ 2 + 0.2 \end{pmatrix}, \quad \dot{\theta}^f = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

show that in this case, by increasing the multirate order, the maneuver is achieved maintaining the performances. More precisely, the multirate order is here 4 on both variables w_1 and w_2. In Figures 8 and 9 the evolution of the two joint variables ψ_1 and ψ_2 and of the orientation θ_2 of the central link are reported.

The discrete time external control is computed on the basis of a multirate order of 4-4, and using a minimization criterion w.r.t. the amplitude of the signals $w_{i,j}$. The so-obtained piecewise external controls are shown in Fig. 10, whereas the applied torques τ_1 and τ_2 are reported in Figs. 11 and 12.

Figure 13 shows the behavior of the Lagrange multiplier λ computed during the first simulation, according to the expression (28). The tolerance value for the simulation was fixed to 10^{-6}. As far as the second simulation is concerned, the same kind of result has been obtained.

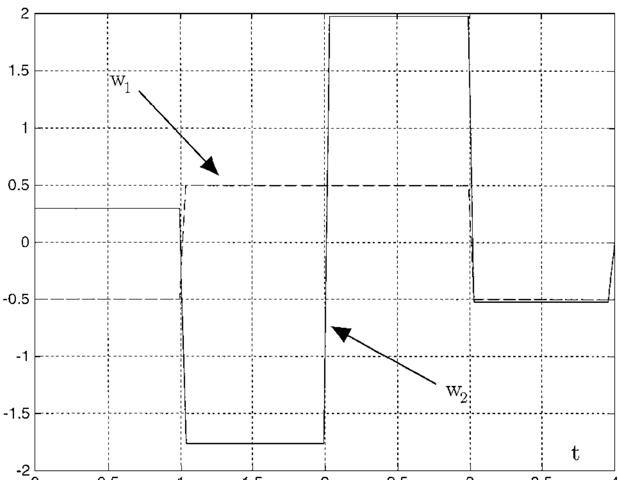

Fig. 10 Piecewise constant external inputs w_1 and w_2.

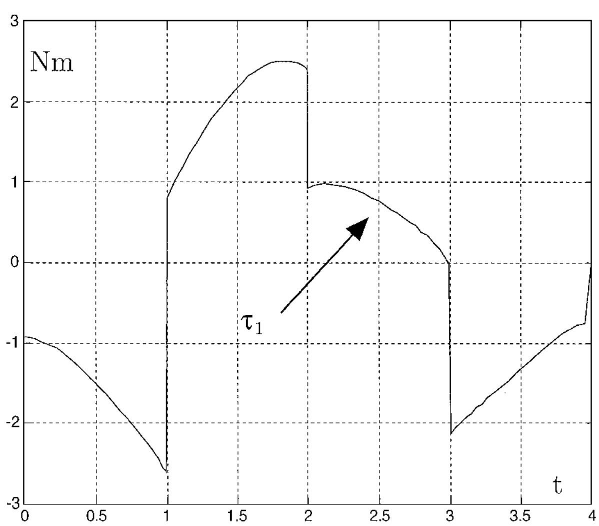

Fig. 11 Applied control torque τ_1.

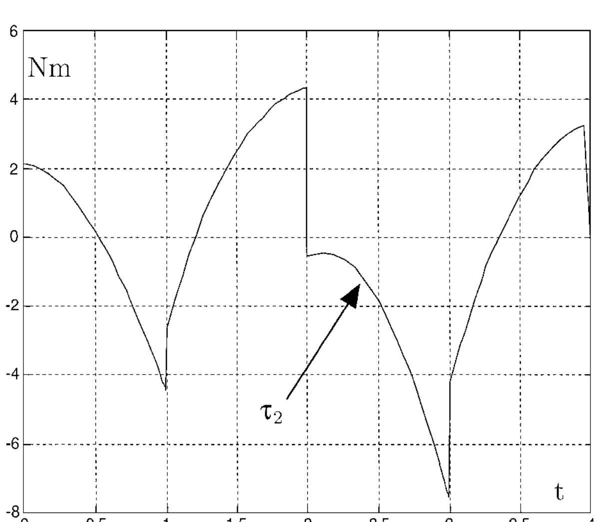

Fig. 12 Applied control torque τ_2.

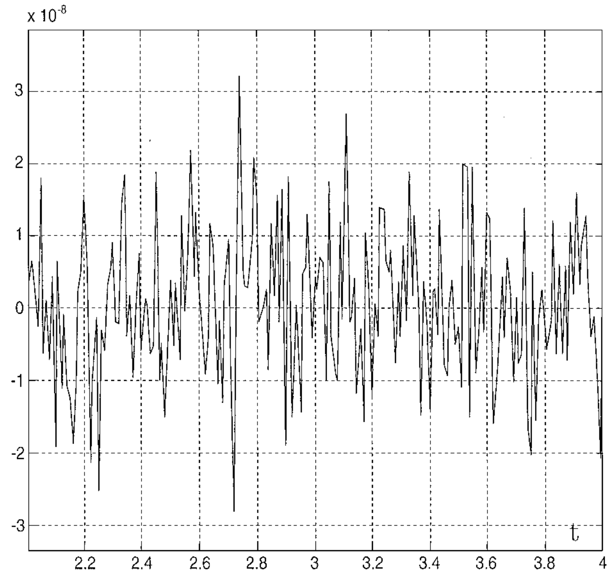

Fig. 13 Behavior of λ during the first simulation.

VI. Conclusions

In this paper a piecewise continuous control law has been designed for attitude and configuration maneuvers of a space multibody structure actuated by internal torques. The relationship between the kinematic and the dynamic model shown in Theorem 1 suggests a simple control design procedure for such a class of systems. First, under the action of a continuous feedback, a finitely sampled dynamics is obtained. Then by inversion arguments, a digital multirate control is computed.

Compared with the inversion-based design technique proposed in Ref. 11, our control strategy admits an iterative implementation viable in presence of perturbations and model uncertainties.[8]

The hybrid scheme proposed here can also be extended to control nonplanar structures because the properties studied in Secs. II–IV still hold for such systems, with an increased computational complexity.

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